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Technical Note

Radiative transfer in one-dimensional hollow cylindrical geometry with anisotropic scattering and variable medium properties

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Abstract

The effects of spatially varying absorption and scattering coefficients on radiation transfer in absorbing, emitting, anisotropically scattering hollow cylinders with reflecting boundaries were investigated using the discrete ordinates method (DOM) by Tsai et al. [Int. J. Heat Mass Transfer 33 (1990) 2651]. Their problem solutions for hollow cylinder cases are incorrect. The cause of this inaccuracies are identified and the correct solutions obtained using DOM S_6 are provided.

Keywords: Hollow cylinder; Radiative transfer; Participating medium; Anisotropic scattering; Discrete ordinates

1. Introduction

The effects of spatially varying absorption and scattering coefficients on radiation transfer in absorbing, emitting, anisotropically scattering, solid and hollow cylinders with reflecting boundaries were investigated using the discrete ordinates method (DOM) by Tsai et al. [1]. Their problem solutions given for hollow cylinders are inaccurate. It is important to correct their results in the literature to aid future researchers in the field. The inaccuracies of the solutions originate from a programming error, and that the correct solutions using DOM S_6 solutions and the exact integral equation solutions for isotropic scattering cases are provided.

2. Problem definition

The development of the equations and the method will be omitted here to save space. However, the same notations used in Ref. [1] will also be used to address the problem which caused erroneous results. In the DOM method, the medium properties such as the extinction $\beta(r)$, scattering $\sigma(r)$, and absorption $\kappa(r)$ coefficients and the anisotropic source $S_m^*(r)$ are evaluated at the node centers. Thus, for equally spaced grids the spatial variable r can be written as $r_{i+1/2} = a_1 + (i - 1/2)\Delta r$ where $\Delta r = (a_2 - a_1)/N$ for $i = 1$ to N, and N is the number of grid spaces used. For solid cylinder, a_1 is set to zero. The error turns out to be in the definition of node-centered spatial locations in which a_1 seems to have been set to zero for hollow cylinder as well. Therefore, in hollow cylindrical medium, the variable medium properties and source are shifted from $a_1 \leqslant r \leqslant a_2$ which is meant to be to $0 \leqslant r \leqslant a_2 - a_1$.

3. Results and discussion

The DOM solutions are basically given by Tsai et al. [1]. The same S_6 quadrature set was used as well. However, in this study, the optical distance is equally

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Nomenclature

a	positions at boundaries
b	optical thickness ($=a_2 - a_1$)
$f(\mu)$	externally incident radiation at boundaries
r	space variable
$I(r,\mu)$	radiation intensity
$S^*(r)$	source function
Greek symbols	
$\beta(r)$	extinction coefficient
$\kappa(r)$	absorption coefficient
μ	direction cosine
$\sigma(r)$	scattering coefficient
Subscripts	
1	position at the inner radius of the hollow sphere
2	position at the outer radius of the hollow sphere
Superscripts	
+	outgoing
−	incoming

divided into 400 grid elements in the DOM solutions. Further grid refinement does not change the results in five decimal places.

In the study, Tsai et al. [1] consider two problems for the hollow cylindrical medium:

Problem 1: The outer shell of the cylinder is subject to unit isotropic external incidence ($f_2(\mu) = 1$). The medium is assumed to have linearly and quadratically varying scattering coefficients. The forward (seven terms), backward (five terms) and isotrotropic scattering cases are considered. The effects of spatially varying scattering coefficient on the hemispherical reflectivity and transmissivity for the case of $a_1 = 1$ and $b = 1$ are investigated. In Table 1, the correct DOM S_6 solutions are given for backward, forward and isotropic scattering alternatives. In the $\sigma(r) = 0.5$ case, the solutions are not affected from the programming error since the scattering coefficient entries are independent of radial position.

Problem 2: The exit distribution of radiation intensity at the inner and outer shells for spherical geometries with $a_1 = 1$, $b = 1$ and 3, and a medium of scattering coefficients of $\sigma(r) = 3r/4F_1$, $\sigma(r) = 0.5$ and $\sigma(r) = 1 - 3r/4F_1$ are investigated. The correct solutions are given in Table 2. In both problems, the reflectivity and the transmissivities are greatly changed from the originally published results.

4. Conclusion

Previously published results by Tsai et al. [1] on the effects of spatially varying absorption and scattering

Table 1
Effects of spatial variation of scattering coefficient, $\sigma(r)$, on hemispherical reflectivity and transmissivity of a hollow cylinder with $a_1 = 1$, $b = 1$, transparent boundaries and $f_2(\mu) = 1\{F_1 = (a_2^3 - a_1^3)/(a_2^2 - a_1^2)$ and $F_2 = (a_2^4 - a_1^4)/(a_2^2 - a_1^2)\}$

$\sigma(r)$	Forward scattering		Isotropic scattering		Backward scattering	
	Reflectivity	Transmissivity	Reflectivity	Transmissivity	Reflectivity	Transmissivity
Linear variation of $\sigma(r)$						
$3r/4F_1$	0.19520	0.44748	0.26249	0.37139	0.27904	0.35358
$0.2 + 9r/20F_1$	0.18651	0.45174	0.25094	0.37364	0.26677	0.35549
$0.4 + 3r/20F_1$	0.17831	0.45621	0.23986	0.37616	0.25497	0.35770
0.5	0.17439	0.45852	0.23449	0.37753	0.24924	0.35890
$0.6 - 3r/20F_1$	0.17058	0.46088	0.22924	0.37897	0.24362	0.36019
$0.8 - 9r/20F_1$	0.16331	0.46579	0.21905	0.38206	0.23270	0.36298
$1 - 3r/4F_1$	0.15646	0.47092	0.20928	0.38547	0.22219	0.36610
Quadratic variation of $\sigma(r)$						
$3r/8F_1 + r^2/2F_2$	0.20612	0.44325	0.27683	0.36932	0.29424	0.35184
$0.4 - 9r/40F_1 + r^2/2F_2$	0.18805	0.45154	0.25306	0.37350	0.26905	0.35535
$0.6 - 21r/40F_1 + r^2/2F_2$	0.17976	0.45599	0.24190	0.37599	0.25717	0.35752
$1 - 9r/8F_1 + r^2/2F_2$	0.16460	0.46553	0.22093	0.38184	0.23474	0.36275

Table 2
Exit distribution of radiation intensity I_m^- at $r = a_1$ and I_m^+ at $r = a_2$ of a hollow cylinder with transparent boundaries and $f_2(\mu) = 1$

b	$I_m^\pm$	m	Variation of scattering coefficient, $\sigma(r), F_1 = (a_2^3 - a_1^3)/(a_2^2 - a_1^2)$								
			Forward scattering			Isotroic scattering			Backward scattering		
			$3r/4F_1$	0.5	$1 - 3r/4F_1$	$3r/4F_1$	0.5	$1 - 3r/4F_1$	$3r/4F_1$	0.5	$1 - 3r/4F_1$
1	$I_m^-(a_1)$	1	0.1522	0.1829	0.2163	0.1194	0.1424	0.1678	0.1104	0.1314	0.1550
		3	0.4288	0.4383	0.4493	0.3538	0.3586	0.3652	0.3360	0.3399	0.3457
		4	0.3117	0.3310	0.3526	0.2567	0.2703	0.2860	0.2430	0.2552	0.2697
		7	0.5457	0.5517	0.5586	0.4552	0.4569	0.4601	0.4347	0.4360	0.4389
		8	0.4935	0.5022	0.5121	0.4112	0.4154	0.4212	0.3921	0.3956	0.4009
		9	0.3797	0.3959	0.4139	0.3160	0.3270	0.3400	0.3004	0.3103	0.3222
	$I_m^+(a_2)$	2	0.3061	0.2551	0.2093	0.3412	0.2869	0.2367	0.3519	0.2955	0.2434
		5	0.4909	0.4373	0.3887	0.5080	0.4533	0.4024	0.5150	0.4587	0.4064
		6	0.0972	0.0840	0.0735	0.1784	0.1564	0.1369	0.1975	0.1734	0.1519
		10	0.6415	0.5959	0.5541	0.6499	0.6039	0.5610	0.6545	0.6073	0.5634
		11	0.2141	0.1855	0.1604	0.2714	0.2355	0.2028	0.2851	0.2472	0.2126
		12	0.0507	0.0508	0.0522	0.1486	0.1389	0.1310	0.1718	0.1604	0.1508
3	$I_m^-(a_1)$	1	0.0155	0.0391	0.0757	0.0084	0.0204	0.0396	0.0069	0.0170	0.0337
		3	0.0724	0.0925	0.1210	0.0409	0.0488	0.0633	0.0355	0.0417	0.0541
		4	0.0493	0.0698	0.1006	0.0279	0.0370	0.0529	0.0240	0.0315	0.0452
		7	0.1347	0.1496	0.1711	0.0809	0.0849	0.0953	0.0727	0.0757	0.0847
		8	0.1165	0.1325	0.1561	0.0698	0.0750	0.0867	0.0626	0.0665	0.0766
		9	0.0838	0.1012	0.1281	0.0502	0.0573	0.0710	0.0448	0.0504	0.0623
	$I_m^+(a_2)$	2	0.3116	0.2040	0.1225	0.3704	0.2494	0.1521	0.3840	0.2588	0.1577
		5	0.4132	0.3064	0.2237	0.4562	0.3396	0.2449	0.4671	0.3468	0.2491
		6	0.1046	0.0588	0.0289	0.2245	0.1472	0.0872	0.2500	0.1672	0.1012
		10	0.5292	0.4330	0.3568	0.5590	0.4556	0.3709	0.5674	0.4609	0.3739
		11	0.1281	0.0632	0.0185	0.2346	0.1424	0.0704	0.2567	0.1594	0.0820
		12	0.0635	0.0461	0.0378	0.2003	0.1447	0.1024	0.2323	0.1706	0.1212

coefficients on radiation transfer in participating, anisotropically scattering hollow cylinder using the DOM contained a programming error. The cause of the error is identified and the corrected solutions using DOM S_6 approximation are given.

Reference

[1] J.R. Tsai, M.N. Özışık, Radiation in cylindrical symmetry with anisotropic scattering and variable properties, Int. J. Heat Mass Transfer 33 (12) (1990) 2651–2658.